PROPAGATION CHARACTERISTICS OF INDUCTIVELY-COUPLED SUPERCONDUCTING MICROSTRIP

J. M. Pond, P. Weaver*, and I. Kaufman†

Microwave Technology Branch
Naval Research Laboratory
Washington, D. C. 20375-5000

Abstract — The propagation properties of two inductively coupled superconducting transmission lines have been studied. An expression for the attenuation of the coupled-line modes was derived for the case of low loss. It was found that, for transmission line geometries of practical interest, the low loss expression agrees well with the more general numerical solution. The numerical solution was used to determine the dispersion characteristics of inductively coupled lines as a function of the superconductor thicknesses and the operating temperature. A lumped-element equivalent circuit, with the same dispersion equation, is presented along with the relationship between the lumped-element values and the physical parameters of the line. Prototype circuit elements such as 20 dB couplers have been designed.

I. INTRODUCTION

Superconducting transmission lines have many advantages for signal processing applications including low loss, wide bandwidth, and low dispersion. Impressive results have been obtained using superconducting microstrip as a basic circuit building block[1]. Recently there has been considerable interest in the miniaturization that can be realized by configuring superconducting microstrip so that the kinetic inductance dominates the magnetic inductance. The kinetic inductance is a measure of the energy stored in the kinetic motion of the charge carriers in a conductor. The effect of this inductance on the phase velocity of a superconducting transmission line was first noted by Pippard[2]. An analytical solution for the propagation characteristics of a superconducting parallel plate transmission line was developed by Swihart[3]. By fabricating a superconducting microstrip where the ground plane, dielectric, and strip are all much thinner than the superconducting penetration depth, phase velocities as low as $0.01c$ have been achieved[4]. In addition to the usual advantages afforded by superconducting electronics, the miniaturization available from this approach could significantly reduce the size and weight of microwave signal processing circuits.

A consequence of achieving the very slow phase velocities is that the superconducting films used for the ground plane and strip are much thinner than the superconducting penetration depth, which is the characteristic decay length of a magnetic field into a superconductor. When the superconducting film is thinner than the penetration depth it is possible for electromagnetic energy on one side of the superconducting film to be transferred to the other side of the film. Owen and Scalapino developed equations describing the transfer of electromagnetic energy (inductive coupling) between a Josephson junction and a superconducting microstrip where one superconducting film is common to both structures and is also thin compared to the penetration depth[5]. This structure has recently been realized and the results agree well with the predicted behavior[6].

Previous work on inductive coupling through a thin lossless superconducting films has concentrated on the transfer of energy between a Josephson junction and a microstrip. In contrast, in the following section the lossy case of two inductively coupled superconducting transmission lines will be considered, where all layers (dielectric and superconductor) can be of arbitrary dimension. In addition to developing a dispersion equation for the two dominant modes of the coupled line, an expression for the attenuation of these two modes is developed and calculations of the loss dependence on several film parameters are made. These results are used to investigate the possibility of exploiting coupled modes for low-loss delay lines and to design prototype circuit elements using inductive coupling as a means of transferring energy between transmission lines.

II. DISPERSION EQUATION

The geometry of the coupled parallel plate transmission lines is shown in Fig 1. The structure consists of a three superconducting layers of thicknesses d_1, d_3, and d_5, respectively. Layers d_1 and d_3 are separated by a dielectric layer of thickness d_2, and layers d_3 and d_5 are separated by a dielectric layer of thickness d_4. It is assumed that only the lowest order TM modes, propagating along the z axis, exist and hence $H_x = H_z = E_y = 0$. It is further assumed that the London equations in local form describe the behavior of the fields in the superconductor and hence the field components in each region ($n = 0,\ldots,6$) are given by

$$\vec{H} = \hat{y}\left\{C_n^- e^{-\gamma_{x_n} x} + C_n^+ e^{+\gamma_{x_n} x}\right\} e^{j(\omega t - k_{z_n} z)} \qquad \text{Eq. 1}$$

where, C_n^- and C_n^+ are unknown amplitudes of the transverse propagating waves. For the superconductors (n = 1, 3, 5)

$$\gamma_{x_n}^2 = k_{z_n}^2 - \omega^2 \mu_0 \varepsilon_n + \lambda_n^{-2} + j\omega\mu_0\sigma_n$$
$$= k_{z_n}^2 - \omega^2 \mu_0 \varepsilon_n + \lambda_n^{-2}\left(1 + j\, 2\lambda_n^2/\delta_n^2\right) \qquad \text{Eq. 2}$$

and, for the dielectric regions (n = 0, 2, 4, 6)

$$\gamma_{x_n}^2 = k_{z_n}^2 - k_n^2 \qquad \text{Eq. 3}$$

where

$$k_n^2 = \omega^2 \mu_0 \varepsilon_0 \varepsilon_{r_n} \qquad \text{Eq. 4}$$

The superconducting penetration depth, λ, is related, via the

* P. Weaver is at the Naval Research Laboratory on sabbatical leave from the University of Hawaii.

† I. Kaufman is with Arizona State University and participated in the ASEE visiting faculty program at the Naval Research Laboratory.

CH2725-0/89/0000-0451$01.00 © 1989 IEEE

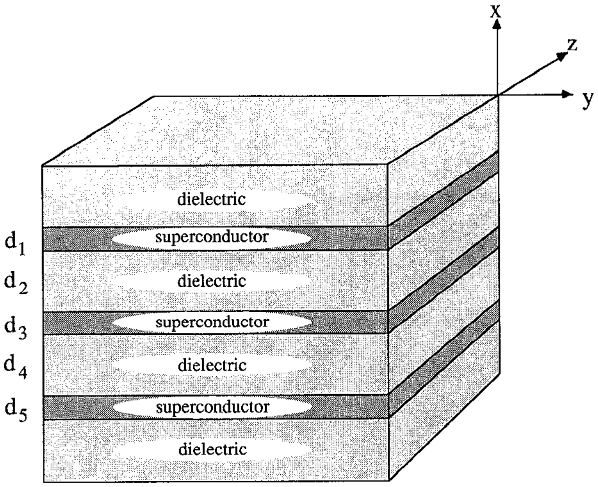

Fig. 1. The geometry of the inductively coupled transmission lines is shown. Each dielectric layer forms a parallel plate waveguide with the two superconducting films which clad it. Coupling is accomplished by the magnetic field penetrating through the center superconductor thus requiring the center superconductor to be on the order of a penetration depth thick.

London equations, to the supercurrent, $\vec{J}_s$, while σ_n is related to the current due to the electrons in the normal state, $\vec{J}_n$. Thus, the total current in the superconductor is given by

$$\vec{J} = \vec{J}_s + \vec{J}_n = \frac{1}{j\omega\,\mu_0\,\lambda_n^2}\vec{E} + \sigma_n\vec{E}$$

$$= \frac{1}{j\omega\,\mu_0\,\lambda_n^2}\left(1 + j\frac{2\lambda_n^2}{\delta_n^2}\right)\vec{E} \quad (n = 1, 3, 5) \quad \text{Eq. 5}$$

The transverse resonance method is applied to determine the dispersion relationship. Under assumptions analogous to those made by Swihart for a parallel plate waveguide, the dispersion relationship becomes

$$k_z^2 = \frac{1}{2}\left\{k_{z_2}^2 + k_{z_4}^2 \pm \left[\left(k_{z_2}^2 - k_{z_4}^2\right)^2 + \frac{4k_2^2k_4^2}{d_2d_4}\left(\frac{\lambda'_3}{\sinh\left(d_3/\lambda'_3\right)}\right)^2\right]^{1/2}\right\} \quad \text{Eq. 6}$$

where

$$k_{z_n}^2 = k_n^2\left\{1 + \frac{\lambda'_{n-1}}{d_n}\coth\left(\frac{d_{n-1}}{\lambda'_{n-1}}\right) + \frac{\lambda'_{n+1}}{d_n}\coth\left(\frac{d_{n+1}}{\lambda'_{n+1}}\right)\right\} \quad (n = 2, 4) \quad \text{Eq. 7}$$

and

$$\lambda'_n = \lambda_n\left(1 + j\,2\left(\lambda_n/\delta_n\right)^2\right)^{-1/2} \quad (n = 1, 3, 5) \quad \text{Eq. 8}$$

The expressions for k_{z_2} and k_{z_4} are the dispersion equations for the uncoupled lines, i.e. when either d_2 or d_4 is infinite, and can be shown to be equivalent to generalized forms of the expression developed by Swihart[3].

Assuming $\delta_n >> \lambda_n$, a first order expansion of Eq. 8, substituted into Eq. 6, yields an expression which is valid for the case of low loss (the dielectric is assumed to be lossless) and is given by

$$k_z^2 = k_{z0}^2 - \frac{j}{2}\left\{k_2^2\Phi_2 + k_4^2\Phi_4 \pm \frac{K_{M1}\left(k_2^2\Phi_2 + k_4^2\Phi_4\right) + K_{M2}^2K_{M3}}{\left(K_{M1}^2 + K_{M2}^2\right)^{1/2}}\right\} \quad \text{Eq. 9}$$

where

$$K_{M1} = k_{z_{2_0}}^2 - k_{z_{4_0}}^2 \quad \text{Eq. 10}$$

$$K_{M2} = \frac{2k_2k_4}{\sqrt{d_2d_4}}\left(\frac{\lambda_3}{\sinh\left(d_3/\lambda_3\right)}\right) \quad \text{Eq. 11}$$

$$K_{M3} = \left(\lambda_3/\delta_3\right)^2\left\{1 + \left(d_3/\lambda_3\right)\coth\left(d_3/\lambda_3\right)\right\} \quad \text{Eq. 12}$$

$$k_{z0} = k_z(\delta_1 = \delta_3 = \delta_5 = \infty) \quad \text{Eq. 13}$$

$$k_{z_{n0}} = k_{z_n}(\delta_{n-1} = \delta_{n+1} = \infty) \quad \text{Eq. 14}$$

$$\Phi_n = \frac{\xi_{n-1}\,\lambda_{n-1}^3}{d_n\,\delta_{n-1}^2}\coth\left(\frac{d_{n-1}}{\lambda_{n-1}}\right) + \frac{\xi_{n+1}\,\lambda_{n+1}^3}{d_n\,\delta_{n+1}^2}\coth\left(\frac{d_{n+1}}{\lambda_{n+1}}\right) \quad (n = 2, 4) \quad \text{Eq. 15}$$

and

$$\xi_n = 1 + \frac{d_n}{\lambda_n}\left[\coth\left(\frac{d_n}{\lambda_n}\right) - \tanh\left(\frac{d_n}{\lambda_n}\right)\right] \quad (n = 1, 3, 5) \quad \text{Eq. 16}$$

The first and second terms of Eq. 9 are the real and imaginary parts, respectively, of k_z^2. Assuming that the real part of Eq. 9 is much larger than the imaginary part of Eq. 9, the real and imaginary parts of k_z are approximately given by

$$\beta_z \equiv Re\left[k_z\right] \cong k_{z0} \quad \text{Eq. 17}$$

$$-\alpha_z \equiv Im\left[k_z\right] \cong Im\left(k_z^2\right)/2k_{z0} \quad \text{Eq. 18}$$

$$\alpha_z\,\lambda_z \cong -\pi\,Im\left(k_z^2\right)/\beta_z^2 \quad \text{Eq. 19}$$

where β_z and α_z are the phase and attenuation constants and λ_z is the wavelength along the transmission line. The loss per wavelength is given by Eq. 19.

The two roots of the dispersion equation correspond to the two possible modes of the transmission line. In Fig. 2 the electric field patterns for the two different modes are shown.

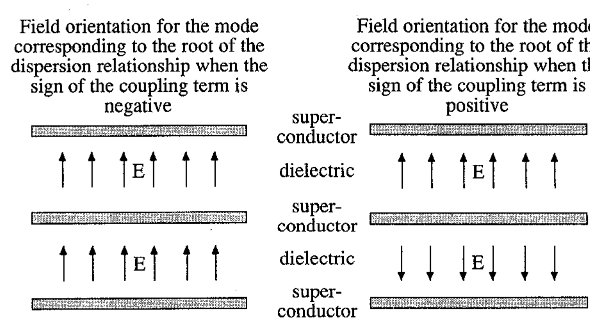

Fig. 2. The electric field orientation for the two dominant modes is shown. The electric fields have the same sign (in phase) when the root of the dispersion relationship uses the positive sign of the coupling term. Similarly, the electric fields have the opposite sign (out of phase) when the root of the dispersion relationship uses the negative sign of the coupling term.

The root given by the dispersion relationship when the coupling term is negative corresponds to the situation where the electric fields are in phase, when the electric fields are arranged out of phase the coupling term is positive. The terms Φ_2 and Φ_4 can be shown to be generalized forms of the loss expression developed by Swihart for an isolated stripline [3].

III. RESULTS

Computations based on these expressions show that the two modes have substantially different propagation constants, especially as the superconducting film thicknesses become substantially smaller than a penetration depth. Throughout the remainder of this discussion, unless otherwise noted, it is assumed that the coupled line structure is symmetric and is composed of superconducting films with a critical temperature of 12 K, a penetration depth of 300 nm, and a normal state conductivity of 1.0 x 10^6 S/m. The dielectrics are assumed to have a relative permittivity of 10 and the operating frequency is 10.0 GHz. These film parameters are consistent with earlier experimental work involving niobium nitride microstrip operating in a kinetic inductance dominated regime [4]. Fig. 3 shows the phase velocity (v_p) and attenuation in dB/wavelength for the symmetric case of $d_1 = d_5 = 20$ nm and $d_2 = d_4 = 100$ nm as the mutual coupling is reduced by varying d_3 from 20 nm to 100 nm. As can be seen from Fig. 3, v_p is substantially

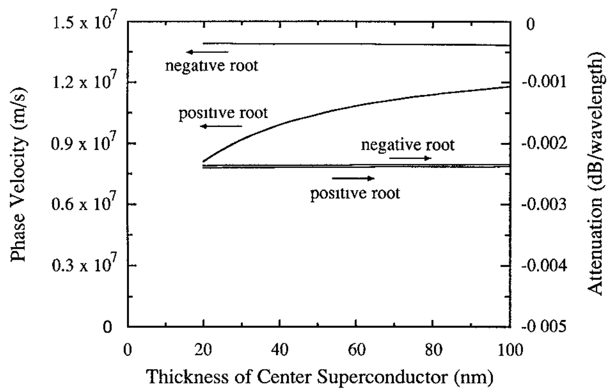

Fig. 3. Phase velocities and attenuations are shown for the two dominant modes of the coupled line as a function of the thickness of the center superconducting film. As the center superconductor thickness increases the coupling decreases.

different for the two modes as d_3 gets thinner. However, the attenuation per wavelength is virtually the same for both cases. This would indicate that for purposes such as delay lines, where a lower v_p yields a more compact component, there is no penalty for using the mode with the lower phase velocity. The temperature dependence of both v_p and the inductive coupling is shown in Fig. 4. Since the penetration depth increases as a function of temperature, v_p decreases for both modes and the coupling increases. The dispersion of these lines is very small at all frequencies and temperatures where the attenuation is reasonable for circuit applications.

An equivalent circuit of distributed elements for the coupled line structure is shown in Fig. 5. The expressions for these elements are determined by comparing Eq. 6 with the dispersion relationship for a pair of magnetically coupled transmission lines. The shunt capacitance, C_n, and shunt conductance, G_n, of each of the dielectric regions (n = 2, 4) follow directly from the parallel plate geometry and the loss properties of the dielectric. In this discussion the losses associated with the dielectric have been ignored, but can easily be included by

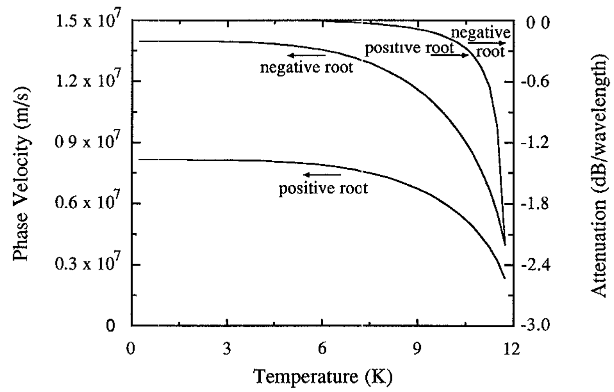

Fig. 4. Phase velocities and attenuations are shown for the two dominant modes of the coupled line as a function of the temperature. As the temperature increases the penetration depth increases resulting in stronger coupling between the transmission lines. Also, as the temperature increases, the density of carriers in the normal state increases causing an increase in the attenuation per wavelength.

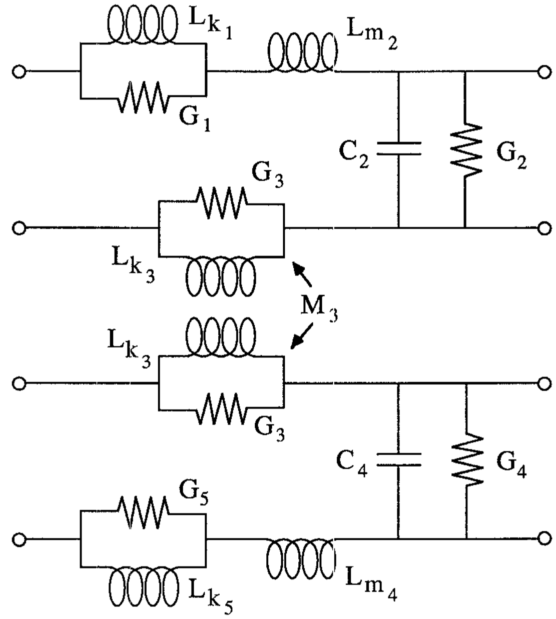

Fig. 5. The equivalent circuit for the coupled line structure is shown. The coupling is modeled by the mutual inductance, M_3, which, to properly account for loss, must be a complex quantity.

allowing ε_2 and ε_4 to be complex permittivities. Each superconductor (n = 1, 3, 5) can be modeled as a parallel combination of its kinetic inductance, L_{k_n}, (accounting for the energy stored due to the carriers in the superconducting state) and its normal conductance, G_n, (accounting for the losses due to the carriers in the normal state) [7]. In addition, the magnetic inductance, L_{m_n} (n = 2, 4), (accounting for the magnetic energy stored in the dielectric regions) has the familiar form for a parallel plate waveguide. The final element is the mutual inductance, M_3, which quantifies the coupling of energy through the common superconducting film. In order to accurately model the dispersion relationship, however, it is necessary for M_3 to be a complex quantity. This is due, in part, to the fact that there are current carriers in both the normal state and superconducting state which contribute to the coupling of energy between transmission lines. The expressions for these quantities are given by

$$C_n = \varepsilon_n \frac{W}{d_n} \qquad (n = 2, 4) \qquad \text{Eq. 20}$$

$$G_n = \sigma_n \frac{W}{d_n} \qquad (n = 2, 4) \qquad \text{Eq. 21}$$

$$L_{m_n} = \mu_0 \frac{d_n}{W} \qquad (n = 2, 4) \qquad \text{Eq. 22}$$

$$L_{m_n} = \mu_0 \frac{\lambda_n}{W} coth\left(\frac{d_n}{\lambda_n}\right) \qquad (n = 1, 3, 5) \qquad \text{Eq. 23}$$

$$G_n = \left(\frac{\sigma_n W}{2}\right) \lambda_n \, tanh\left(\frac{d_n}{\lambda_n}\right)\left\{1 + \frac{d_n}{\lambda_n}\left[coth\left(\frac{d_n}{\lambda_n}\right) - tanh\left(\frac{d_n}{\lambda_n}\right)\right]\right\} \qquad (n = 1, 3, 5) \qquad \text{Eq. 24}$$

$$M_3 = \mu_0 \frac{\lambda_3}{W} csch\left(\frac{d_3}{\lambda_3}\right)\left\{1 - j\left(\frac{d_3}{\lambda_3}\right)^2\left[1 + \frac{d_3}{\lambda_3} coth\left(\frac{d_3}{\lambda_3}\right)\right]\right\} \qquad \text{Eq. 25}$$

where W is the width of the parallel plate waveguide. The expressions for the capacitances, conductances, and inductances are given on a per unit length basis. This circuit model makes it easy to determine, using conventional microwave computer-aided-design software, the response of more complicated circuit topologies which utilize inductively coupled structures.

A design for a simple half-wavelength forward coupler which transfers a signal with a coupling factor of 20 dB from one superconducting transmission line to another could be fabricated by simply overlaying two microstrip transmission lines for a precisely determined length. The total amount of coupling is then determined primarily by the interaction length and the thickness of the common superconductor relative to its penetration depth. In particular, at 10.0 GHz a coupled signal of −20 dB requires an interaction length of 0.6 mm for a symmetric structure where $d_1 = d_5 = 16$ nm, $d_2 = d_4 = 100$ nm, and $d_3 = 210$ nm. Substantially stronger coupling occurs for thinner values of d_3. In fact, it is often necessary to use strong coupling approximations for coupler design when the common superconducting film is significantly less than a penetration depth thick.

IV. Conclusions

A dispersion relationship has been developed for the coupled transmission line case of three arbitrarily thick superconductors separated by two arbitrarily thick dielectric layers. The dispersion equation, which was found by the transverse resonance method, can be used to solve for the propagation characteristics of the two dominant coupled line modes. These results have been used to show that, in the symmetric case, the mode with the lower phase velocity has the same attenuation per wavelength as the mode with the higher phase velocity. In addition, a first order expansion of the dispersion relationship yields an accurate closed-form expression for the attenuation of the coupled line modes. Using a mutual inductance to describe the coupling, an equivalent circuit has been presented, which accurately models the propagation characteristics of the coupled transmission line modes. The equivalent circuit model facilitates the design of circuits using inductively coupled structures as has been shown with the design of a 20 dB coupler. Experiments to verify these concepts have been initiated.

Acknowledgment

The authors would like to thank Drs. H. Newman, D. Webb and C. Rauscher for their comments and suggestions.

References

[1] R. S. Withers, A. C. Anderson, P. V. Wright, and S. A. Reible, "Superconductive tapped delay lines for microwave analog signal processing," *IEEE Trans. Magn.*, vol. MAG-19, no. 3, pp. 480-484, May 1983.

[2] A. B. Pippard, "The surface impedance of superconductor and normal metals at high frequencies, III. The relationship between impedance and superconducting penetration depth," *Proc. Roy. Soc. Lond.*, vol. A191, pp. 399-415, 1947.

[3] J. C. Swihart, "Field solution for a thin-film superconducting strip transmission line," *J. Appl. Phys.*, vol. 32, no. 3, pp. 461-469, March. 1961.

[4] J. M. Pond, J. H. Claassen, and W. L. Carter, "Measurements and modeling of kinetic inductance microstrip delay lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, no.12, pp. 1256-1262, December. 1987.

[5] C. S. Owen and D. J. Scalapino, "Inductive coupling of Josephson junctions to external circuits", *J. Appl. Phys.*, vol. 41, no. 5, pp. 2047-2056, April 1970.

[6] K. Yoshida, T. Nagatsuma, S. Kumataka, and K. Enpuku, "Millimeterwave emission from Josephson oscillator through thin film junction electrode", *IEEE Trans. Magn.*, vol. MAG-23, no. 2, pp. 1283-1286, March 1987.

[7] T. VanDuzer and C. W. Turner, *Principles of Superconductive Devices and Circuits*. New York: Elsevier, 1981.